

# Mid-term

July 18, 2019

**Problem 1:** Prove that any 1-d piecewise constant function defined on  $[-2,2]$  (w.r.t. polyhedrons), say, constant in  $[-2,-1)$ ,  $[-1,0)$ ,  $[0,1)$  and  $[1,2]$ , can be written as a DNN function with the 1-d Heaviside activation function. The Heaviside activation function is

$$H(a) = \begin{cases} 0, & a \leq 0, \\ 1, & a > 0. \end{cases}$$

**Problem 2:**

a. Given an array of integers where each element is between 1 and  $n$  ( $n = \text{length of array}$ ), some elements appear twice and others appear once.

Find all the elements between 1 and  $n$  that do not appear in this array.

e.g. Input=  $[4,3,2,7,8,2,3,1]$ , output=  $[5,6]$

b. For a number between 1 and 100, print the product of its prime factors.

e.g. Input=60, output=  $'60=2*2*3*5'$ .

**Problem 3:** First, load the MNIST dataset as follows. Take the first 5000 ( $[:5000]$ ) as training set, the following 1000 ( $[5000:6000]$ ) as validation set, and the following 1000 ( $[6000:7000]$ ) as test set. Then construct a fully connected neural network with one hidden layer, with the hidden layer width being 500. Choose your own loss function and optimization method. Finally, tune the learning rate to get the best possible test accuracy.