Homework 4

July 19, 2019

Due: 7.22 24:00
Homework email: psupku2019@163.com
Please include your name

Theoretical part

Problem 1: Find a bilinear function on rectangular τ_i, s.t. $\phi(x_k, x_l) = \phi_{kl}$, $(k,l) \in \{(i,j), (i+1,j), (i,j+1), (i+1,j+1)$.

Problem 2: Prove that DNN_1 is dense in $C(\Omega)$ for any σ, that is Riemann integrable.

Problem 3: Compute
$8u_{ij} - (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} + u_{i-1,j-1} + u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1}) = f_{ij}$

Problem 4 (Optional): Find largest $\eta_0 > 0$ s.t. this gradient descent method converge if $\eta < \eta_0$. $u^{k+1} = u^k - \eta(f - A * u^k)$.

Problem 5: Apply gradient descent method twice,

$$u^{k+\frac{1}{2}} = u^k - \eta(f - A * u^k) \tag{1}$$

$$u^{k+1} = u^{k+\frac{1}{2}} - \eta(f - A * u^{k+\frac{1}{2}}). \tag{2}$$

Prove that if $\eta = \frac{1}{16}$,

$$u^{k+1} = u^k - S * (f - A * u^k),$$

where $S = \frac{1}{256}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 24 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

Problem 6(Optional): The dual basis exists uniquely.